Mandelbrot Set

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| Name: Mofazzal Hossain.  ID: 193021042  Department of CSE.  Primeasia University.  City :Kishoreginj ,Dhaka,Bangladesh.  Email :mofazzalhossain91966@gmail.com | Name: Md Alif Hossain.  ID: 193009042  Department of CSE.  Primeasia University.  City :Kishoregonj,Dhaka,Bangladesh.  Email :alifakonda@gmail.com |
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| **Introduction:** One of the most stunning and well-known mathematical images of all time is the Mandelbrot set, named after mathematician Benoit Mandelbrot, who contributed greatly to its study. An image of the Mandelbrot set is shown below:    **Figure 1: The Mandelbrot Set**  **Iteration of the Mandelbrot set function:**  The formal definition of the Mandelbrot set is the set of values **c** for which a starting seed of 0 remains bounded upon iteration of thefunction **+1 = + c** (Lei, 2000). On first glance, this definition definitely appears confusing, and so it is necessary to clarify some terms.  Iteration simply means the procedure of repeating a process over and over again. In the context of mathematics and the above definition, this process is the utilization of a mathematical function. And as mentioned, the specific function we will be iterating is  **+1 = + c**, where **c** is a constant (unspecified at the moment). Beginning with a **seed**, the first value to which the function is applied, we can iterate, using the result of each iteration as the new input for the next calculation:  **+1 = + c**       **+1 = + c**← Inserting the seed, **z0**, to receive an output of **z1**  **= + c**← z**1**is used as the new input, and we iterate again  **= + c**← The process continues...  **= + c …..**  **z3 = -1**  It can be observed that this orbit cycles between the two values of **0**and **-1**; or in other words, this orbit has a **period** of **2**.  Various **c** values have drastically different orbits, as shown in the created histogram graphs below. The plots shown represent orbits for **+1 = + c**   where **c = 0.2**, **-1.75**, and **-1.85**, with the seed still remaining as **0**. For **c**= **0.2**, the orbit tends to a fixed point, converging fairly quickly. For **c** = **-1.75**, the orbit tends to a period of **3**. However, for **c**= **-1.85**, the orbit does not appear to conform to any recognizable pattern - we can describe this orbit as **chaotic.**  **Construction of the Mandelbrot Set Image:**  With the definition of the Mandelbrot set and the complex plane clarified, the construction of the famous Mandelbrot set image can easily be understood. Simply consider an area of the complex plane at the origin. A massive grid of points is overlaid on the area, and a computer checks whether each point is part of the Mandelbrot set or not, by testing whether the correspondent orbit attached to each point escapes to infinity or not. If the point is part of the  **Escape Criterion**:  Consider a circle of radius **2**centered at the origin. If the orbit of any particular **c**value ever leaves this circle, the orbit will escape to infinity and the **c** value is not part of the Mandelbrot set (Devaney, 2006).   Now that we have a definition for what it means to escape to infinity, we can define how long any orbit takes to escape - namely, the number of iterations until the orbit leaves the circle. This allows us to color the points based on the **escape time** of each corresponding orbit. An example of such an image is shown below, with the color bands clearly defined: | Many have seen and admired the beauty of this image, and but very few people, including myself prior to undertaking this essay, understand and appreciate the mathematics behind this complex fractal image. Fractals have always intimidated me, with the dizzying spirals and repeating patterns, and so I chose to investigate this topic in hopes of gaining more understanding in this area of mathematics. The purpose of this essay is to examine the definition of the Mandelbrot set and also how its image is constructed. In addition, I aim to analyze and discern notable patterns in this remarkable image. Most importantly, I hope to gain a greater appreciation of the beauty of this image and communicate my findings in an easily accessible manner. However, in order to comprehend the mathematics behind the Mandelbrot set, we must grasp several other concepts first.  This process continues, generating a list of values as we iterate. This list of values can be referred to as the **orbit** of **z0**. Now, what happens when a value is chosen for **c**? How will the orbit behave? These questions can be answered by exploring several examples. Suppose the constant **1** is chosen for **c**. For a seed of **0**:  **= + c =** + 1**= 1**  **= + c=** + 1**= 2**  **= + c=** + 1**= 5**  **= + c =** + 1**= 26**  **= + c =** + 1**= 677**  **…**  It is clear that this orbit will tend to infinity. Choosing another value, **c = 0**, while allowing the seed to remain at **0**, we can see that the orbit remains constant:  **z0 = 0**  **z1 = 0**  **z2 = 0**   If another value, **c = -1**, is chosen, with the seed still remaining at **0**, another pattern emerges:  **z0 = 0**  **z1 = -1**  **z2 = 0**  Mandelbrot set, it is colored black; if not, it is colored white. A completed image may look like the following:  **mandel2.bmp**  **Figure 2: Yellow and Blue**  **Mandelbrot Set Image**  **Escape Criterion**:  Consider a circle of radius **2**centered at the origin. If the orbit of any particular **c**value ever leaves this circle, the orbit will escape to infinity and the **c** value is not part of the Mandelbrot set (Devaney, 2006).   Now that we have a definition for what it means to escape to infinity, we can define how long any orbit takes to escape - namely, the number of iterations until the orbit leaves the circle. This allows us to color the points based on the **escape time** of each corresponding orbit. An example of such an image is shown below, with the color bands clearly defined:  **mandel4.bmp**  **Figure 3: Mandelbrot Set with**  **Defined Color Stripes**  **Conclusion:**  Through the course of this essay, we explored the definition of the Mandelbrot set and how iteration is used to define the set, as well as the various fates different orbits can take. We explored the use of the complex plane and the construction of the Mandelbrot set image, as well as the possibilities of coloring the image. |
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